C12 Specimen (MA)

Q1a) $(25x^4)^{\frac{1}{2}} = \boxed{5x^2}$

b) $(25x^4)^{-3/2} = \dfrac{1}{(25x^4)^{3/2}} = \dfrac{1}{(\sqrt{25x^4})^3} = \dfrac{1}{(5x^2)^3}$

$= \boxed{\dfrac{1}{125x^6}}$

Q2) $(3-x)^6 \simeq 3^6 + \binom{6}{1}(3)^5(-x)^1 + \binom{6}{2}(3)^4(-x)^2$

$\simeq \boxed{729 - 1458x + 1215x^2}$

Q3i) $(5-\sqrt{8})(1+\sqrt{2}) = 5 + 5\sqrt{2} - \sqrt{8} - \sqrt{16}$

$= 5 - 4 + 5\sqrt{2} - 2\sqrt{2}$ $\qquad \sqrt{8} = 2\sqrt{2}$

$= \boxed{1 + 3\sqrt{2}}$

ii) $\sqrt{80} + \dfrac{30}{\sqrt{5}} = \sqrt{16 \times 5} + \dfrac{30 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$

$= 4\sqrt{5} + \dfrac{30\sqrt{5}}{5} = 4\sqrt{5} + 6\sqrt{5} = \boxed{10\sqrt{5}}$

Q4a) $\dfrac{dy}{dx} = 10x^4 - 3x^{-4} = \boxed{10x^4 - \dfrac{3}{x^4}}$

b) $\int (y)\,dx = \int [2x^5 + 7 + x^{-3}]\,dx = \left[\dfrac{2x^6}{6} + 7x + \dfrac{x^{-2}}{-2} + c\right]$

$= \dfrac{x^6}{6} + 7x - \dfrac{1}{2x^2} + c$

5) $h = \frac{b-a}{n} = \frac{3-2}{4} = \frac{1}{4}$ //

$\therefore$ Area $\simeq \frac{1}{2} \times \frac{1}{4}\left[0.5 + 0.2 + 2[0.379 + 0.299 + 0.242]\right]$

$\simeq \boxed{0.318}$

6a) $\underline{f(1) = 7}$: $1 + 1 + 2 + a + b = 7$

$a + b = 7 - 4$

$\underline{\underline{a + b = 3}}$.

b) $\underline{f(-2) = -8}$: $(-2)^4 + (-2)^3 + 2(-2)^2 - 2a + b = -8$

$16 \cancel{-8} + 8 - 2a + b = \cancel{-8}$

$\underline{\underline{2a - b = 24}}$.

from (a), $b = 3 - a$: $2a - (3 - a) = 24$

$3a - 3 = 24$

$3a = 27$ $\therefore \boxed{a = 9}$

$\therefore$ $b = 3 - 9 = \boxed{-6 = b}$

Q7a) $a_2 = 3a_1 - c = 3(2) - c = \boxed{6 - c}$

b) $a_1 + a_2 + a_3 = 0$.

$a_1 = 2$

$a_2 = 6 - c$

$a_3 = 3(6 - c) - c = 18 - 4c$

$\therefore a_1 + a_2 + a_3 = 2 + 6 - c + 18 - 4c = 0$

$\Rightarrow 26 - 5c = 0$

$c = \frac{26}{5} = \boxed{5.2}$

8a) $(k+3)x^2 + 6x + k = 5$

$\Rightarrow (k+3)x^2 + 6x + (k-5) = 0$

$\Rightarrow$ $a = (k+3)$, $b = 6$, $c = (k-5)$ two distinct real roots $\therefore b^2 - 4ac > 0$

$b^2 - 4ac > 0$

$(6)^2 - 4(k+3)(k-5) > 0$

$36 - 4(k^2 - 2k - 15) > 0$

$9 > k^2 - 2k - 15$

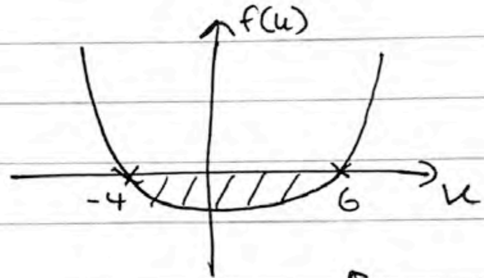
$\therefore k^2 - 2k - 24 < 0$

b) finding critical values

$k^2 - 2k - 24 = 0$

$(k-6)(k+4) = 0$

$k = 6,\ k = -4$



we want where $k^2 - 2k - 24 < 0$
[ie where 'y-values' < 0 on the graph]

so range : $\boxed{-4 < k < 6}$

9a) taking log of both sides : (base 3)

$\log_3(y) = \log_3(3x^2)$

$\log_3(y) = \log_3(3) + \log_3(x^2)$ power rule.

$\log_3(y) = 1 + 2\log_3(x)$

b) $1 + 2\log_3 x = \log_3(28x - 9)$

$1 + \log_3(x^2) = \log_3(28x - 9)$

$1 = \log_3(28x - 9) - \log_3(x^2)$

$\log_3\left(\frac{28x - 9}{x^2}\right) = 1$

$\therefore 3^1 = \frac{28x - 9}{x^2}$

$\therefore 3x^2 = 28x - 9$

$3x^2 - 28x + 9 = 0$

$(3x - 1)(x - 9) = 0$

$\therefore \boxed{x = \frac{1}{3}}$ and $\boxed{x = 9}$

<u>alt</u>: LHS $= \log_3(y) = \log_3(28x - 9)$

$\Rightarrow \quad y = 28x - 9$

$x = \sqrt{\frac{y}{3}} \quad \Rightarrow \quad y = 28\sqrt{\frac{y}{3}} - 9$

$(y + 9) = 28\sqrt{\frac{y}{3}}$

$(y + 9)^2 = 784\left(\frac{y}{3}\right)$

$y^2 + 18y + 81 = \frac{784y}{3}$

$\therefore y^2 - \frac{730}{3}y + 81 = 0$ //

By Quadratic Formula: $y = 243$, $y = \frac{1}{3}$

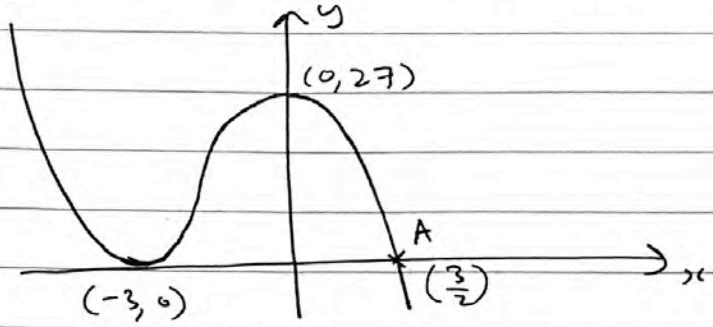
$x = \sqrt{\frac{y}{3}} \quad \therefore x = \sqrt{\frac{243}{3}} \quad x = \sqrt{\frac{\frac{1}{3}}{3}}$

$\boxed{x = 9}$ $\boxed{x = \frac{1}{3}}$

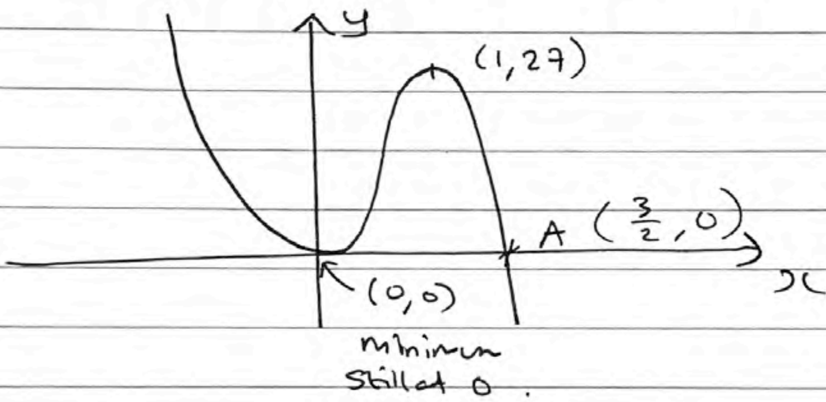
10a) $\underline{y=0}$: $9 - 2x = 0$

$x = \boxed{\frac{9}{2}}$ $\therefore A\left(\frac{9}{2}, 0\right)$

b i) $f(x+3)$ = translation in the -ve x-dir by 3.



ii) $f(3x)$ = multiply all x-coords' by $\frac{1}{3}$.



c) $f(x)$ must be shifted down by 17 units to have a minimum at $\overline{(3, 10)}$

$\therefore$ $\boxed{k = -17}$

11a) $x + 4 = -x^2 + 2x + 24$

$$x^2 + x - 2x + 4 - 24 = 0$$
$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$
$$x = 5, \quad x = -4.$$

$$\left.\begin{array}{l}\text{at } x=5,\ y = 5+4 = 9 \\ \text{at } x=-4,\ y=0.\end{array}\right\} \therefore \begin{array}{l} A(-4, 0) \\ B(5, 9)\end{array}$$

b) $$R = \int_{-4}^{5} (y_2 - y_1)\,dx = \int_{-4}^{5} \left[-x^2 + 2x + 24 - x - 4\right] dx$$

$$= \int_{-4}^{5} \left[-x^2 + x + 20\right] dx = \left[-\frac{x^3}{3} + \frac{x^2}{2} + 20x\right]_{-4}^{5}$$

$$= \left[\frac{-125}{3} + \frac{25}{2} + 100\right] - \left[\frac{64}{3} + 8 + (-80)\right]$$

$$= \frac{425}{6} + \frac{152}{3} = \boxed{121.5}$$

12a) $\text{radius} = |AB| = \sqrt{(10-2)^2 + (7-1)^2} = 10$ //

$$\therefore \underline{(x-2)^2 + (y-1)^2 = 100}$$

b) the tangent will be perpendicular to the line AB.

$$M_{AB} = \frac{7-1}{10-2} = \frac{6}{8} = \frac{3}{4} //$$

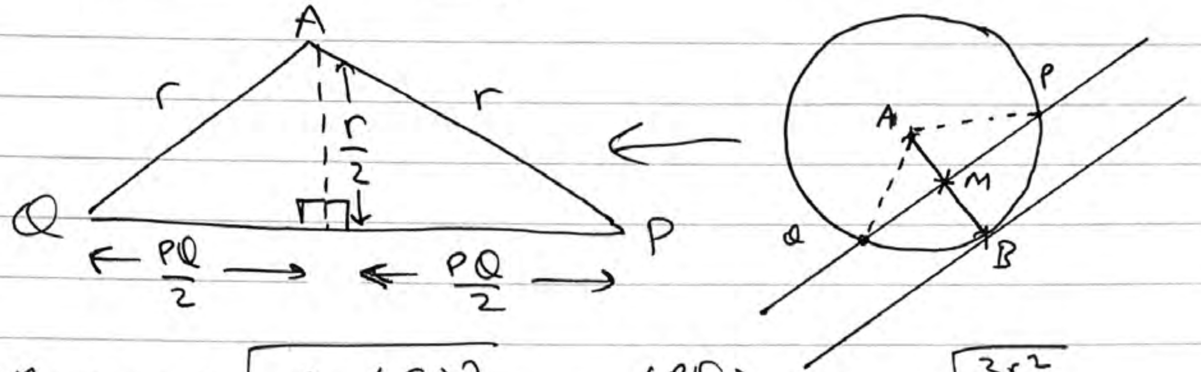
$$\therefore M_{TANGENT} = -\frac{4}{3} // \quad \left(-\frac{4}{3} \times \frac{3}{4} = -1\right)$$

$\Rightarrow \quad y - 7 = -\frac{4}{3}(x - 10)$

$\Rightarrow \quad y = -\frac{4}{3}x + \frac{40}{3} + 7$

$\Rightarrow \quad \boxed{y = -\frac{4}{3}x + \frac{61}{3}}$

c)

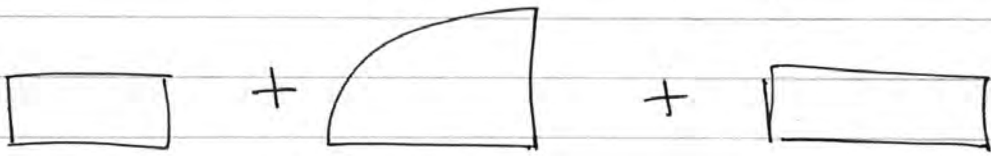


Pythagoras : $\sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \left(\frac{PQ}{2}\right) = \sqrt{\frac{3r^2}{4}}$ //

$\therefore PQ = 2\sqrt{\frac{3r^2}{4}}$

$r = 10 : \quad PQ = 2\sqrt{\frac{3(100)}{4}} = 2\sqrt{75} = \boxed{10\sqrt{3}}$

13a)



Area: $\left| \; xy + \frac{\pi x^2}{4} + xy \; \right| = 4$

$\Rightarrow \quad 2xy + \frac{\pi x^2}{4} = 4$ //

$\Rightarrow \quad 2xy = 4 - \frac{\pi x^2}{4}$

$\Rightarrow \quad y = \frac{4 - \frac{\pi x^2}{4}}{2x} = \boxed{\frac{16 - \pi x^2}{8x}}$

× top & bottom by 4

b) $P = x + y + y + y + x + y + r\theta$

$= 2x + 4y + x\left(\frac{\pi}{2}\right)$

$= 2x + 4\left(\frac{16 - \pi x^2}{8x}\right) + \frac{\pi x}{2}$

$= x\left(2 + \frac{\pi}{2}\right) + \frac{16 - \pi x^2}{2x}$

$= 2x + \cancel{\frac{x\pi}{2}} + \frac{8}{x} - \cancel{\frac{\pi x}{2}} = \boxed{2x + \frac{8}{x}}$

c) $P = 8x^{-1} + 2x$

$\frac{dP}{dx} = -8x^{-2} + 2 = 0$ //

$\Rightarrow 2 = \frac{8}{x^2}$

$\Rightarrow 2x^2 = 8 \quad \therefore x^2 = 4$

$\Rightarrow x = 2$ // (reject $x = -2$)

$x > 0$.

check: $\frac{d^2P}{dx^2} = 16x^{-3} = \frac{16}{x^3} > 0$ (for all $x > 0$).

$\therefore$ P is minimum at $x = 2$.

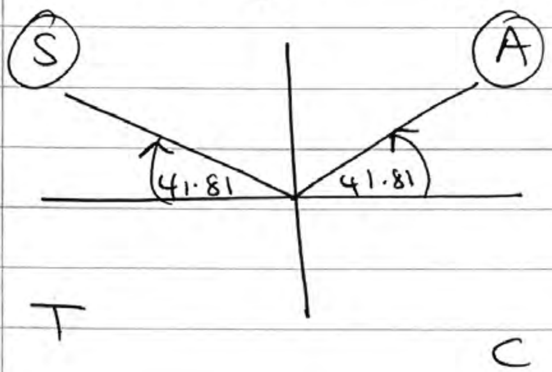
$P = \frac{8}{2} + 2(2) = \boxed{8\text{cm}}$

14a) $3\sin(x+45) = 2$

$\sin(x+45) = \frac{2}{3}$.

$\sin^{-1}\left(\frac{2}{3}\right) = x+45 = 41.81^\circ$

new range: $\underline{45 \leq x+45 \leq 405^\circ}$



$x+45^\circ = (180-41.81), (360+41.81)$

$x+45^\circ = 138.2^\circ, 401.8^\circ$

$\therefore \boxed{x = 93.2^\circ, 356.8^\circ}$

b) $2\sin^2 x + 2 = 7\cos x$

$2 - 2\cos^2 x + 2 = 7\cos x$

$2\cos^2 x + 7\cos x - 4 = 0$

$(2\cos x + 1)(\cos x - 4) = 0$

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

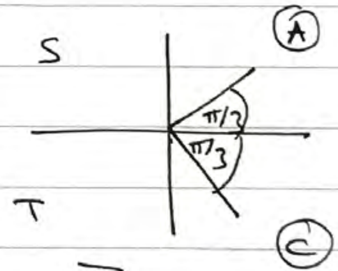
$\cos x - 4 = 0$

$\cos x = 4$ x reject.

no valid solution

$x = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3}$ //

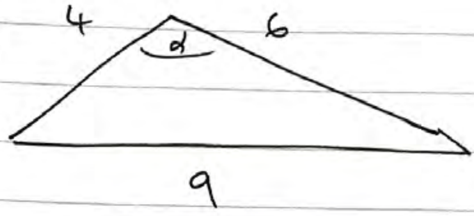
solving in: $\boxed{0 \leq x \leq 2\pi}$



$x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\boxed{x = \frac{\pi}{3}^c, \frac{5\pi}{3}^c}$

15a)



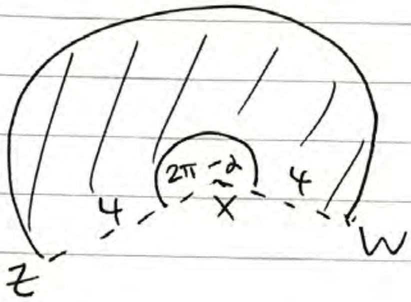
cosine rule

$$\cos\alpha = \frac{6^2+4^2-9^2}{2(6)(4)} = \frac{-29}{48}$$

$$\alpha = \cos^{-1}\left(\frac{-29}{48}\right) = 2.2195\ldots$$

$$= \boxed{2.22^c}$$

b)



$2\pi - \alpha = 2\pi - 2.22$

$\therefore$ Area required $= \frac{1}{2}(4^2)(2\pi - 2.22)$

$= \boxed{32.5\text{ cm}^2}$

c) Area $\Delta XZY = \frac{1}{2}ab\sin C = \frac{1}{2}(4)(6)\sin(2.22)$

$= 9.5588\ldots\text{ cm}^2.$

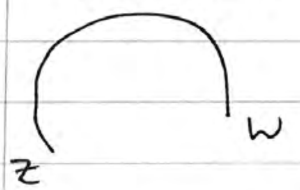
Area from (b) $= 32.5\text{ cm}^2$

$\therefore$ Shaded area $= 32.5 + 9.56 = \boxed{42.06\text{cm}^2}$

$=$ 42.1cm^2 to 3sf.

d) length $WY = 6 - 4 = 2\text{cm}$

arc $ZW = r\theta = 4\times(2\pi - 2.22) = 16.25\text{cm}$



length $ZY =$ ~~[illegible]~~ 9 cm.

$\therefore$ Perimeter $= 2 + 16.25 + 9 = \boxed{27.3\text{cm}}$

16a) $\sum_{1}^{17}(1.5) = 1.5\times 17 = \boxed{25.5\text{km}}$

b) $a + a+d + a+2d$; 1.5, 1.75, 2.00 $\Big\}$ $a = 1.5$, $d = 0.25$

17^{th} term $= a+(17-1)d = 1.5+16(0.25) = \boxed{5.5\text{km}}$

c) $a + ar + ar^2 + ar^3 + \ldots$; 1.5, $1.5\times(1.05)$, $1.5\times(1.05)^2 \ldots$ $\Big\}$ $a = 1.5$, $r = 1.05$

$$S_{17} = \frac{a(1-r^n)}{1-r} = \frac{1.5(1-(1.05)^{17})}{1-1.05} = \boxed{38.8\text{km}}$$

d) For running: $S_{17} = \frac{n}{2}[2a+(n-1)d] = \frac{17}{2}[3+16(0.25)]$

$= 59.5\text{km}$

$\therefore$ Total distance $= 59.5+25.5+38.8$

$= \boxed{123.8\text{km}}$

e) we want $ar^{N-1} > 40$.

$a = 1.5$, $r = 1.05$ $\Big\}$ $1.5(1.05)^{N-1} > 40$

$$(1.05)^{N-1} > \frac{80}{3}$$

$$\log(1.05)^{N-1} > \log\left(\frac{80}{3}\right)$$

$$(N-1) > \frac{\log(80/3)}{\log(1.05)} \quad \therefore N-1 > 67.297\ldots$$

$\therefore N > 68.297\ldots$

So the 69^{th} day is when she first cycles > 40km.